The 2-element interferometer

Fundamentals of Radio Interferometry: Chapter 4, part 1/3

Griffin Foster (& Julien Girard)
SKA-SA/Rhodes University

NASSP 2016

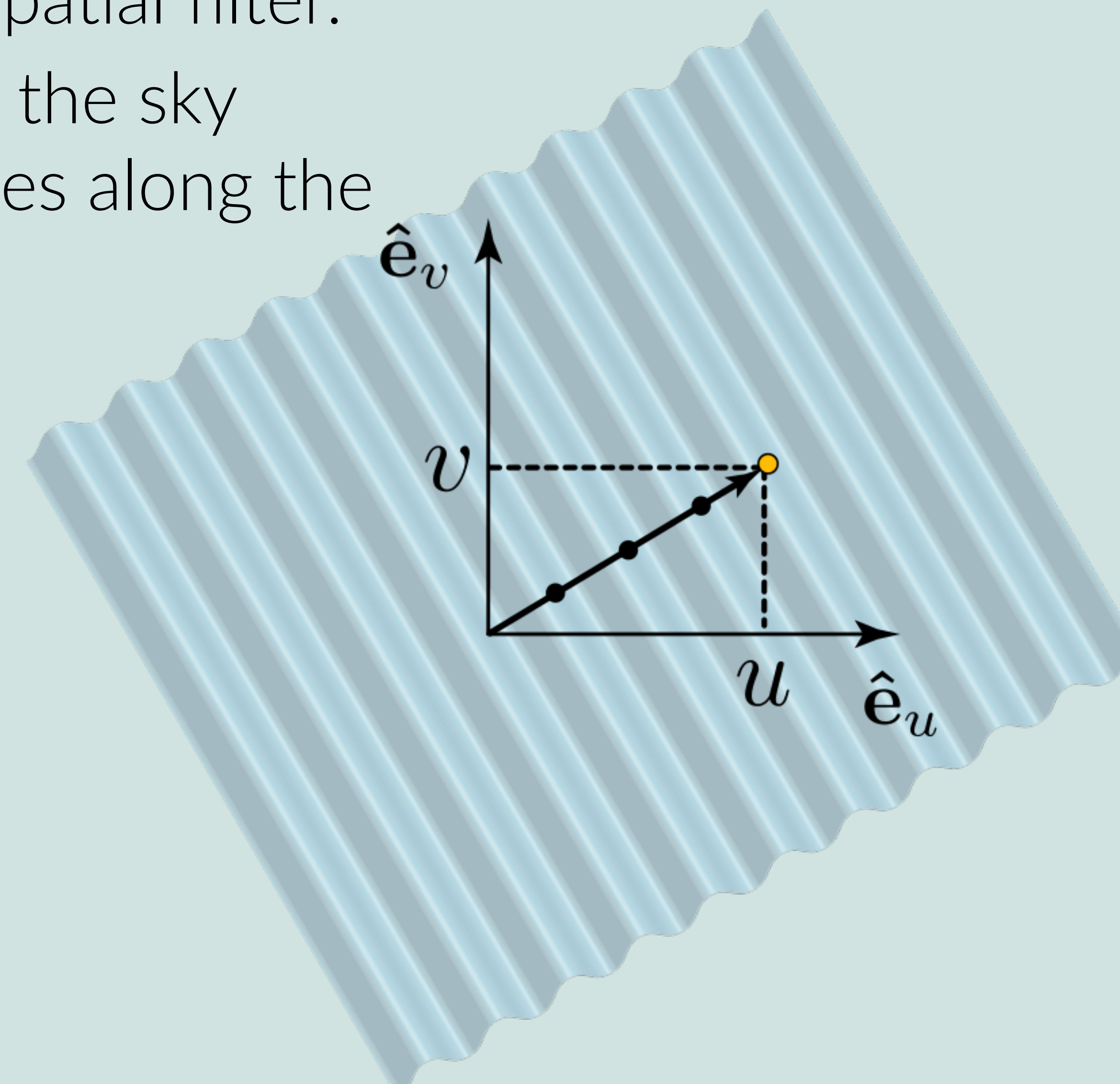
The 2-element interferometer : a 2D spatial filter

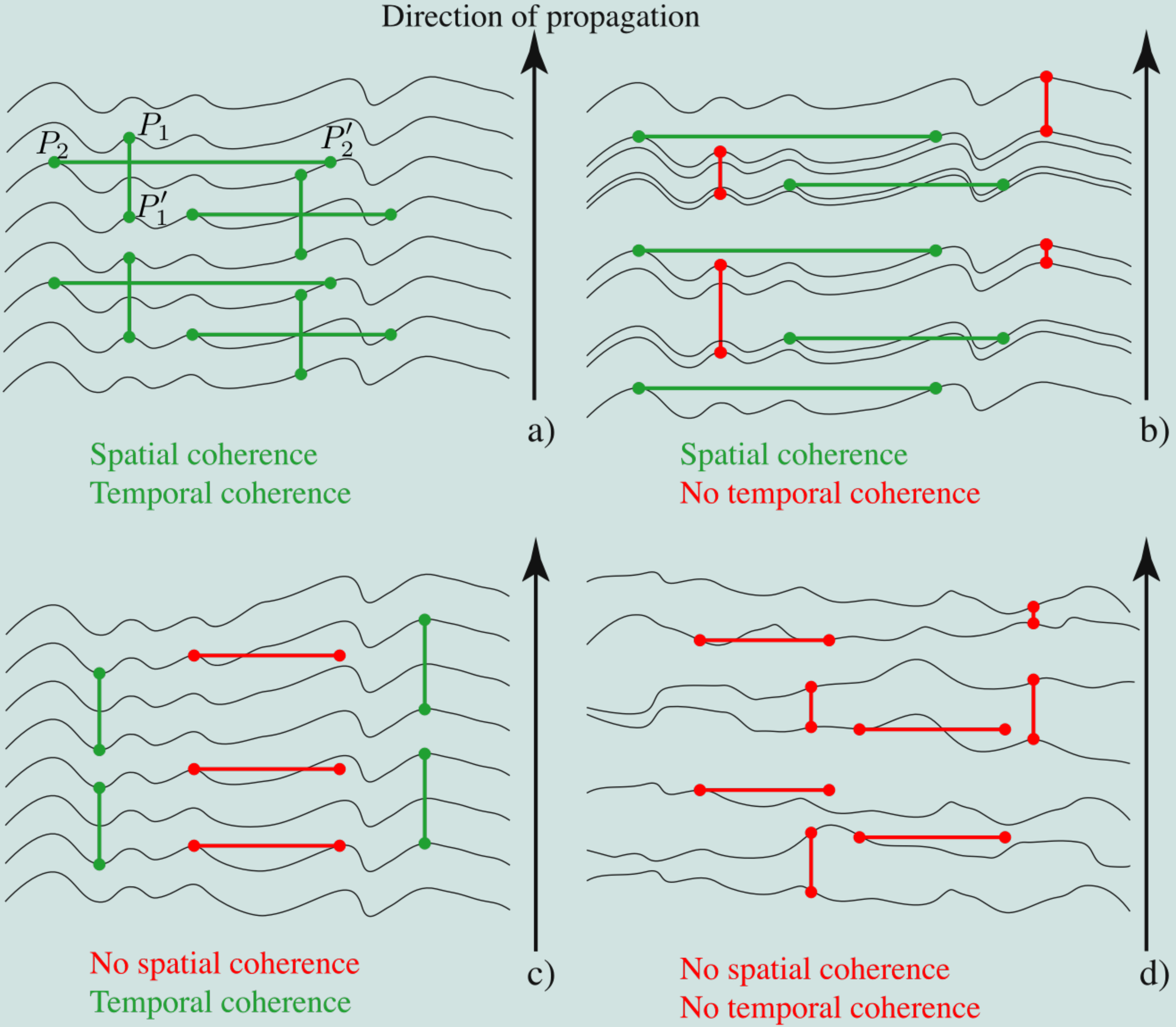
$$\mathbf{f}_{u,v}^{l,m} = e^{-2j\pi(ul+vm)}$$

$$r_{uv} = \sqrt{u^2 + v^2}$$

The Fourier kernel acts as a spatial filter.

If (l,m) are the coordinates of the sky (u,v) are the spatial frequencies along the same axes.





Direction of propagation
P_1
P_2
P_2'
P_1'
a)
Spatial coherence
Temporal coherence
b)
Spatial coherence
No temporal coherence
c)
No spatial coherence
Temporal coherence
d)
No spatial coherence
No temporal coherence

The 2-element interferometer : the “receiving” case

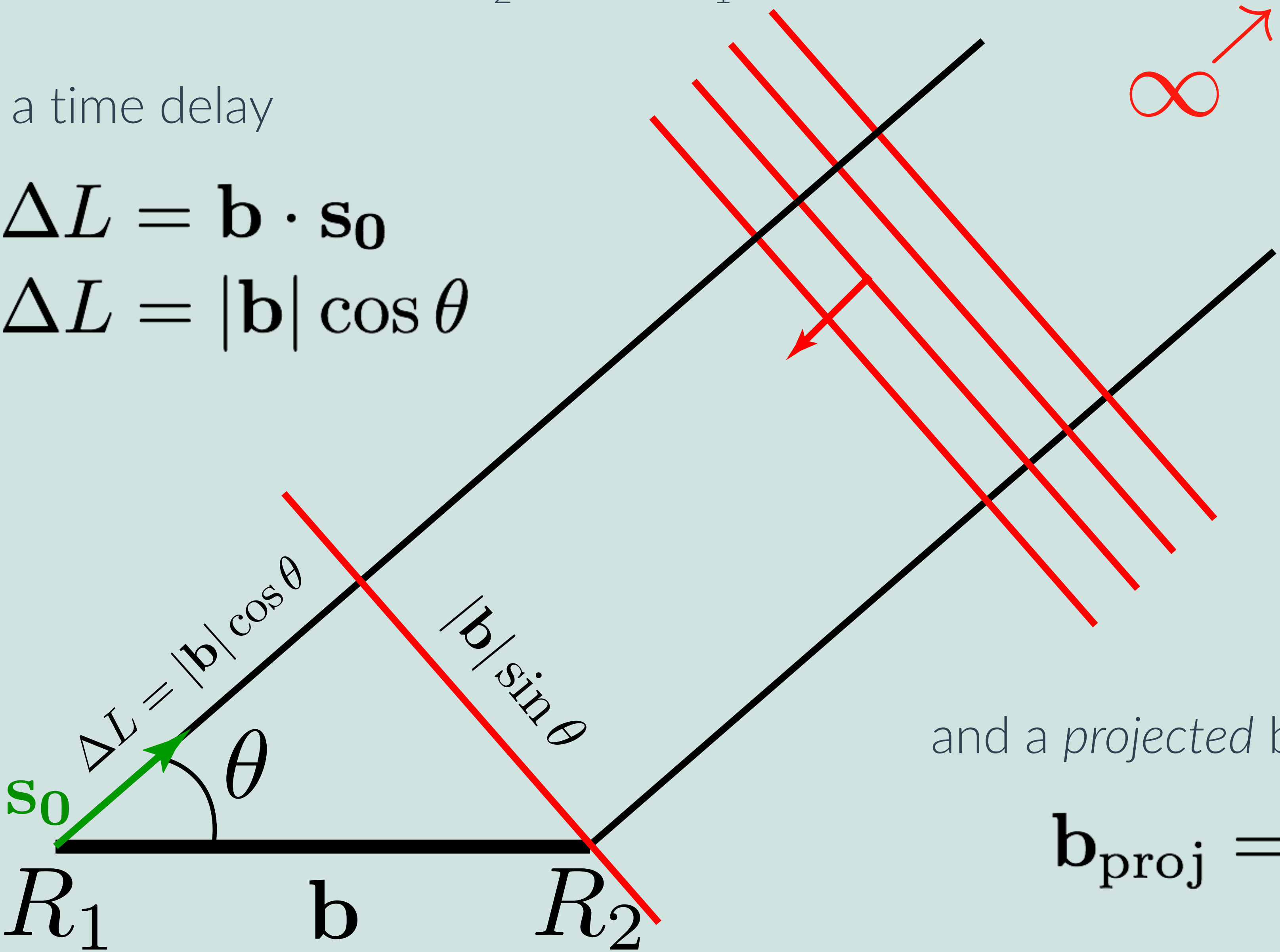
Let’s now consider 2 receivers illuminated by a plane wave from θ

The signal will reach R_2 before R_1 and creates:

a time delay

$$\Delta L = \mathbf{b} \cdot \mathbf{s_0}$$

$$\Delta L = |\mathbf{b}| \cos\theta$$



and a *projected* baseline

$$\mathbf{b}_{\mathbf{proj}} = |\mathbf{b}| \sin\theta$$

The 2-element interferometer : the “receiving” case

Let V_1 and V_2 the measured voltages at R_1 and R_2

$$V_1 = V_{01}\cos(\omega t + \varphi_1) \quad V_2 = V_{02}\cos(\omega t + \varphi_2)$$

The 2-element interferometer : the "receiving" case

Let V_1 and V_2 the measured voltages at R_1 and R_2

$$V_1 = V_{01} \cos(\omega t + \varphi_1) \quad V_2 = V_{02} \cos(\omega t + \varphi_2)$$

We use $\mathbf{s_0}$ as a reference direction of the incoming wave

The 2-element interferometer : the "receiving" case

Let V_1 and V_2 the measured voltages at R_1 and R_2

$$V_1 = V_{01}\cos(\omega t + \varphi_1) \quad V_2 = V_{02}\cos(\omega t + \varphi_2)$$

We use $\mathbf{s_0}$ as a reference direction of the incoming wave

R_1 as the reference receiver, shifting the origin of time so that $\varphi_1 = 0$

We can rewrite the expression of the voltages as

$$V_1 = V_{01}\cos(\omega t), \quad V_2 = V_{02}\cos(\omega t + \varphi_2 - \varphi_1)$$

The 2-element interferometer : the “receiving” case

Let V_1 and V_2 the measured voltages at R_1 and R_2

$$V_1 = V_{01}\cos(\omega t + \varphi_1) \quad V_2 = V_{02}\cos(\omega t + \varphi_2)$$

We use $\mathbf{s_0}$ as a reference direction of the incoming wave

R_1 as the reference receiver, shifting the origin of time so that $\varphi_1 = 0$

We can rewrite the expression of the voltages as

$$V_1 = V_{01}\cos(\omega t), \quad V_2 = V_{02}\cos(\omega t + \varphi_2 - \varphi_1)$$

$\varphi_2 - \varphi_1$ is linked to ΔL the extra path required for the plane

wave to go from R_2 to R_1.

The 2-element interferometer : the “receiving” case

Let V_1 and V_2 the measured voltages at R_1 and R_2

$$V_1 = V_{01}\cos(\omega t + \varphi_1) \quad V_2 = V_{02}\cos(\omega t + \varphi_2)$$

We use $\mathbf{s_0}$ as a reference direction of the incoming wave

R_1 as the reference receiver, shifting the origin of time so that $\varphi_1 = 0$

We can rewrite the expression of the voltages as

$$V_1 = V_{01}\cos(\omega t), \quad V_2 = V_{02}\cos(\omega t + \varphi_2 - \varphi_1)$$

$\varphi_2 - \varphi_1$ is linked to ΔL the extra path required for the plane

wave to go from R_2 to R_1.

$$\varphi_2 - \varphi_1 = \Delta\Phi = \frac{\omega}{c}\Delta L \quad \text{with} \quad \Delta L = \mathbf{b}\cdot\mathbf{s_0}$$

The 2-element interferometer : The Π interferometer

The product of two signals can be implemented through the correlation operation.

The 2-element interferometer : The Π interferometer

The product of two signals can be implemented through the correlation operation.

A correlator is a device which multiplies two voltages

$$C = \langle V_1 V_2 \rangle_t$$

The 2-element interferometer : The Π interferometer

The product of two signals can be implemented through the correlation operation.

A correlator is a device which multiplies two voltages

$$C = \langle V_1 V_2 \rangle_t$$

$$C = \langle V_{01} V_{02} \cos \omega t \cos \left[\omega (t + \frac{\Delta L}{c})\right] \rangle_t$$

$$\langle V_{01} V_{02} \cos \omega t \cos \left[\omega (t + \tau)\right] \rangle_t$$ with $$\tau = \frac{\Delta L}{c}$$

The 2-element interferometer : The Π interferometer

The product of two signals can be implemented through the correlation operation.

A correlator is a device which multiplies two voltages

$$C = \langle V_1 V_2 \rangle_t$$

$$C = \langle V_{01} V_{02} \cos \omega t \cos [\omega (t + \frac{\Delta L}{c})] \rangle_t$$

$$\langle V_{01} V_{02} \cos \omega t \cos [\omega (t + \tau)] \rangle_t$$ with $$\tau = \frac{\Delta L}{c}$$

$$C = V_0^2 \frac{\langle \cos(2\omega t + \tau) + \cos(\omega \tau) \rangle_t}{2}$$

The 2-element interferometer : The Π interferometer

To reduce the level of noise, the correlator performs some averaging in time.

We will assume that this averaging time is long enough so that the fast temporal oscillations caused by ωt is smoothed out.

$$C = V_0^2 \frac{\langle \cos(2\omega t + \tau) + \cos(\omega\tau) \rangle_t}{2}$$

The 2-element interferometer : The Π interferometer

To reduce the level of noise, the correlator performs some averaging in time.

We will assume that this averaging time is long enough so that the fast temporal oscillations caused by ωt is smoothed out.

$$C = V_0^2 \frac{\langle \cos(2\omega t + \tau) + \cos(\omega\tau) \rangle_t}{2}$$

slowly variable

The 2-element interferometer : The Π interferometer

To reduce the level of noise, the correlator performs some averaging in time.

We will assume that this averaging time is long enough so that the fast temporal oscillations caused by ωt is smoothed out.

$$C = V_0^2 \frac{\langle \cancel{\cos(2\omega t + \tau)} + \cos(\omega\tau) \rangle_t}{2}$$

slowly variable

It is equivalent to filter the signals with a low-pass filter which role is to remove the fast-varying component of the signal. We call the remaining quantity the correlation given by a cosine correlator.

$$C_{\cos} = \frac{V_0^2}{2} \cos \omega\tau$$

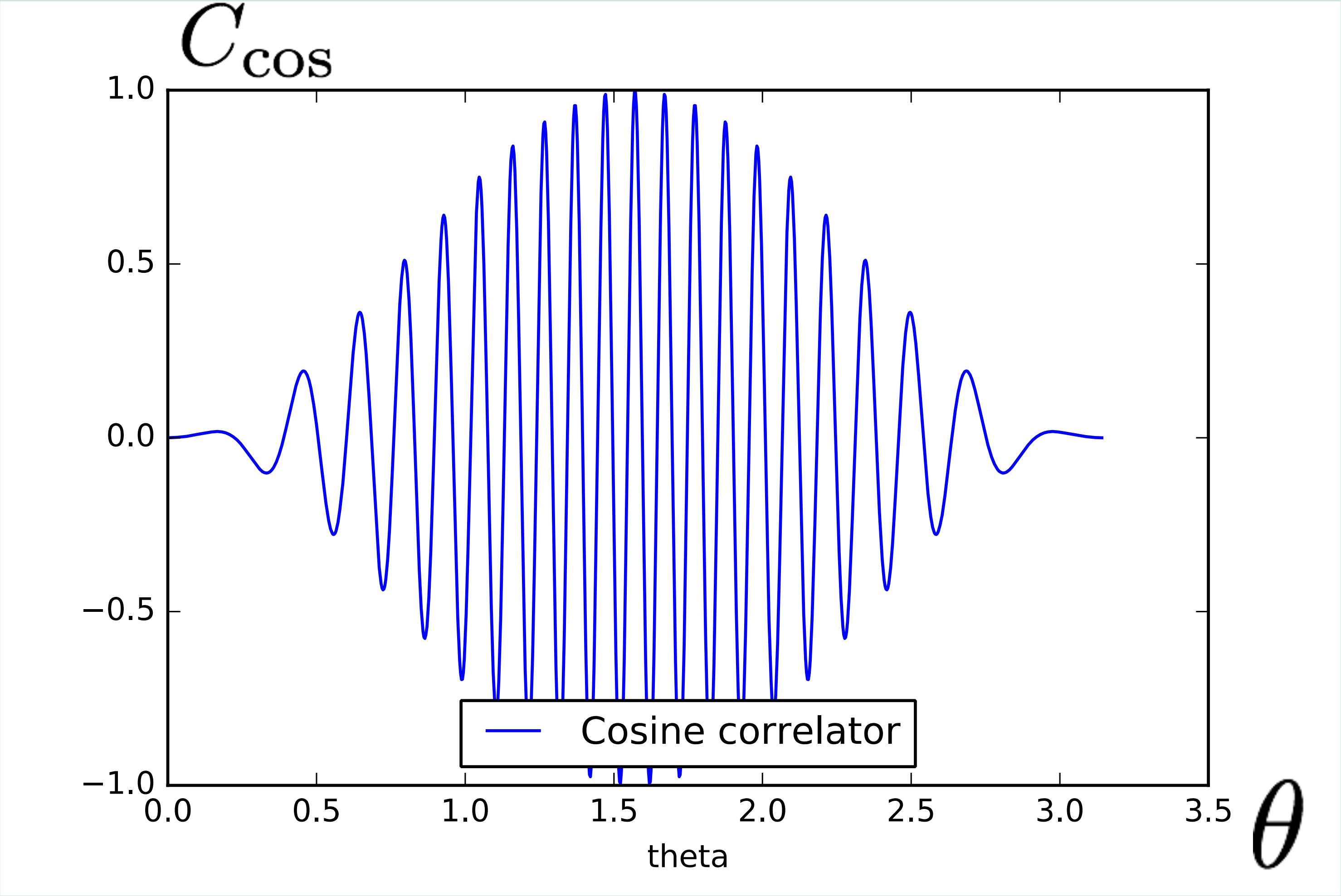
The 2-element interferometer : The Π interferometer

$$C_{\cos} = \frac{V_0^2}{2} \cos \omega\tau$$

This correlation depends on the delay τ

It describes a fringe pattern in the sky

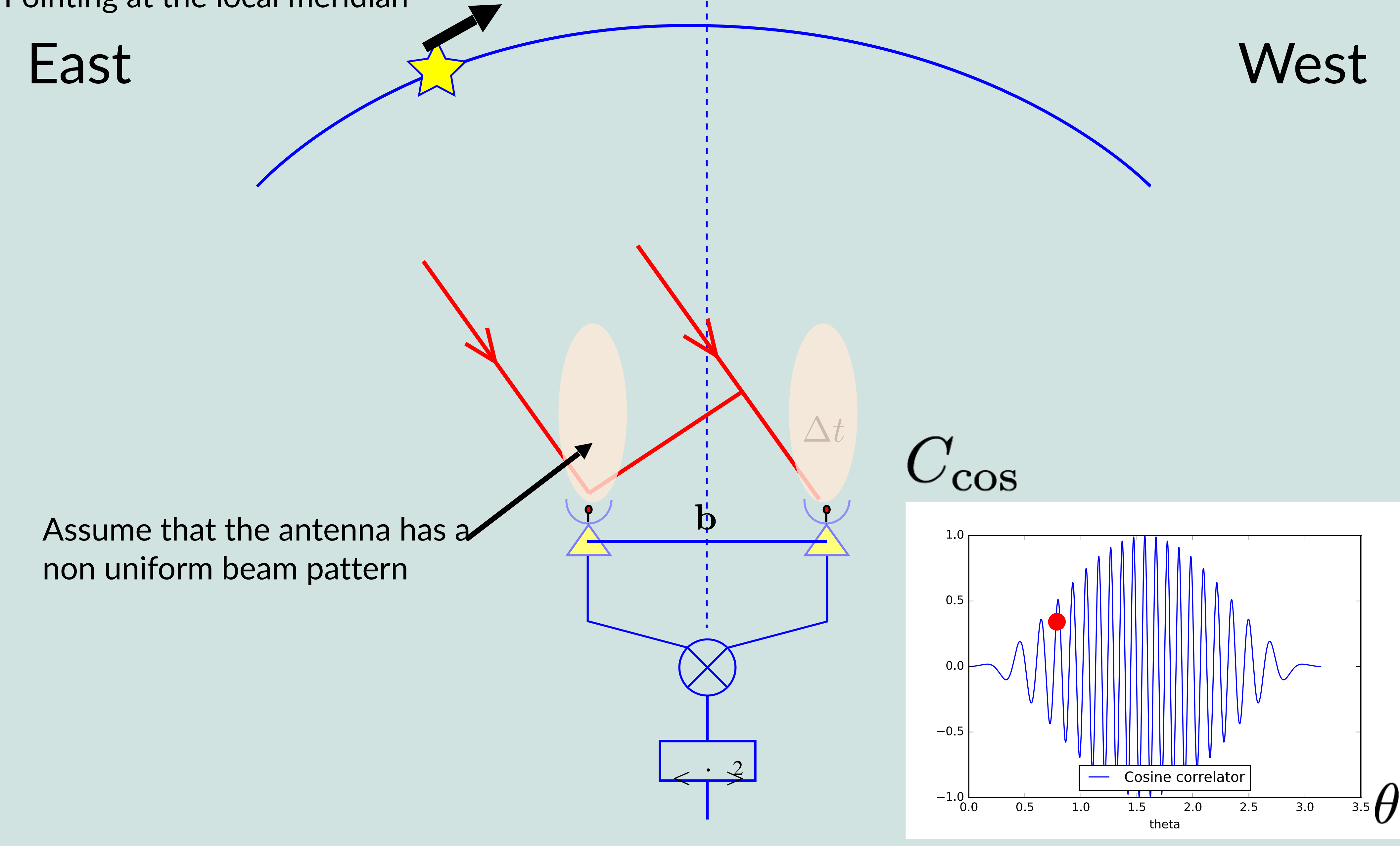
Which can be modulated for example by the antenna pattern



The 2-element interferometer : Untracked source

Let’s assume an East-West interferometer

Pointing at the local meridian



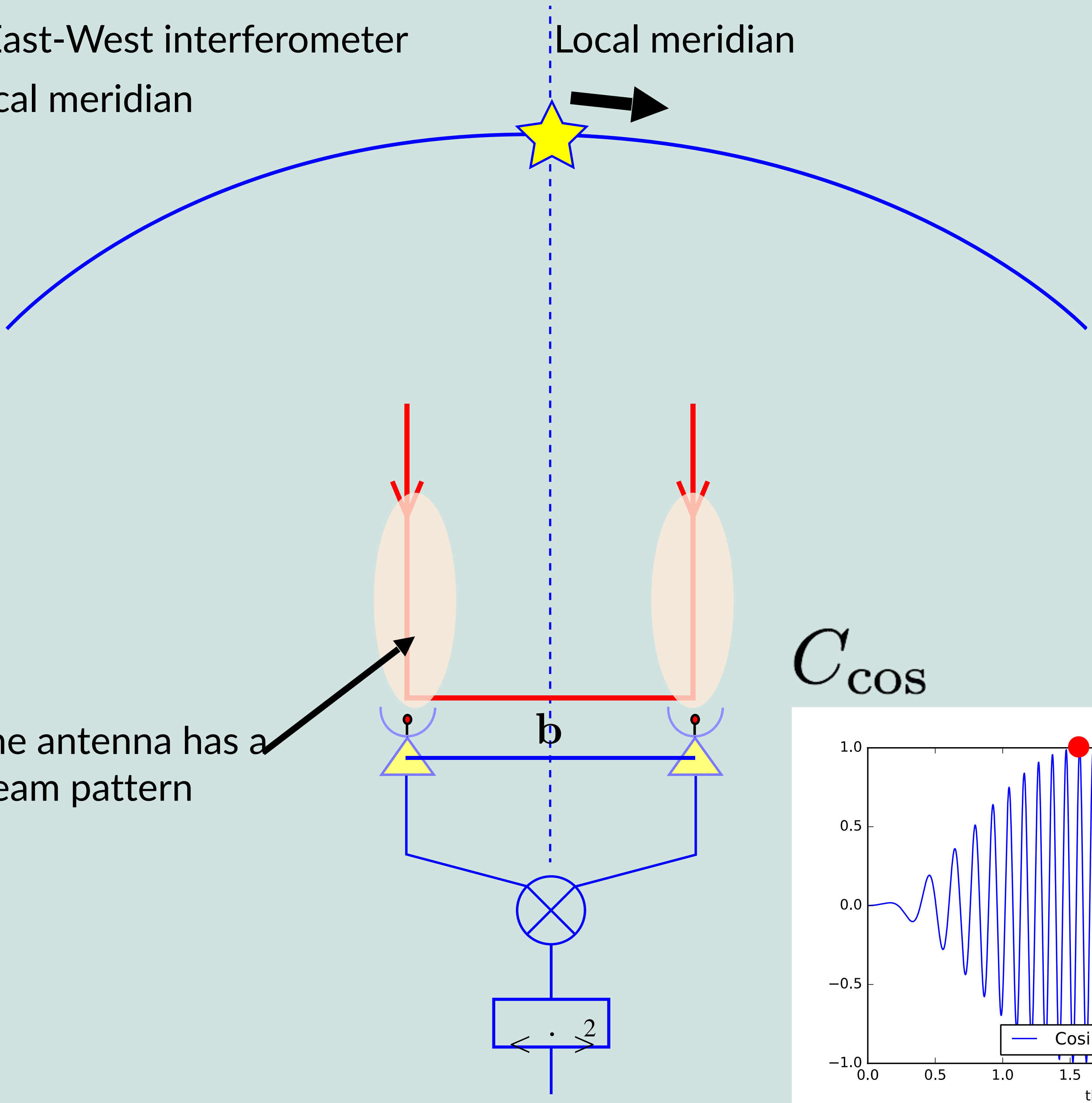
The 2-element interferometer : Untracked source

Let's assume an East-West interferometer

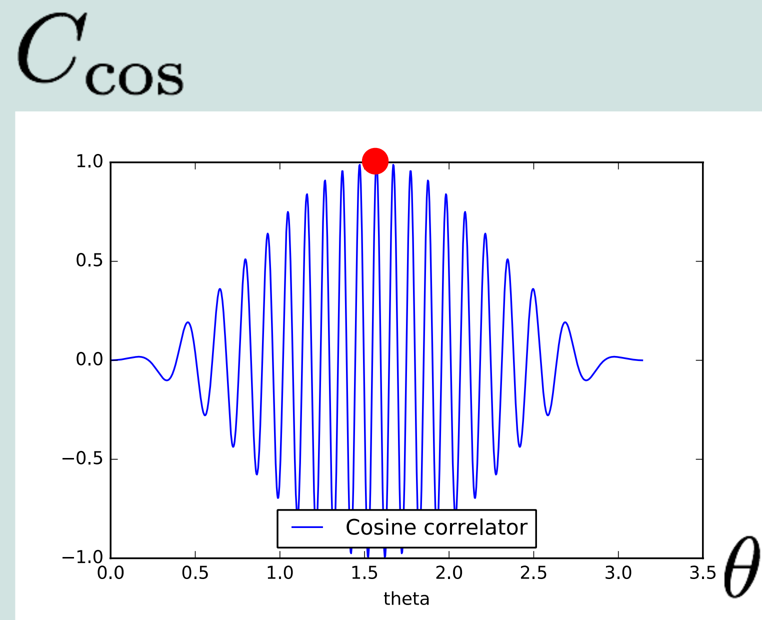
Pointing at the local meridian

East

West



Assume that the antenna has a non uniform beam pattern



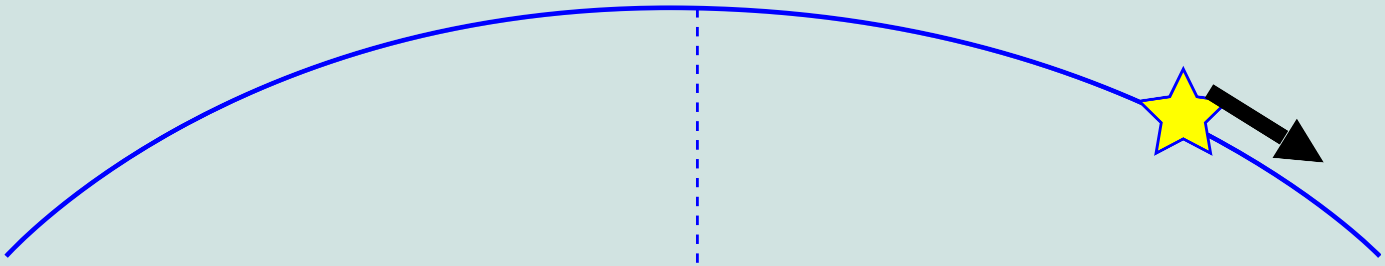
The 2-element interferometer : Untracked source

Let's assume an East-West interferometer

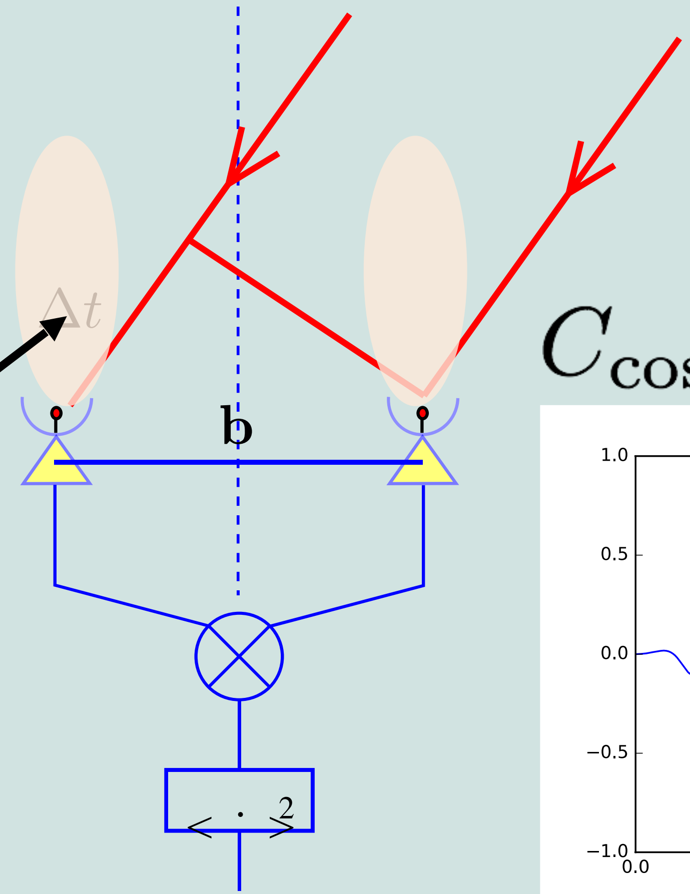
Local meridian

Pointing at the local meridian

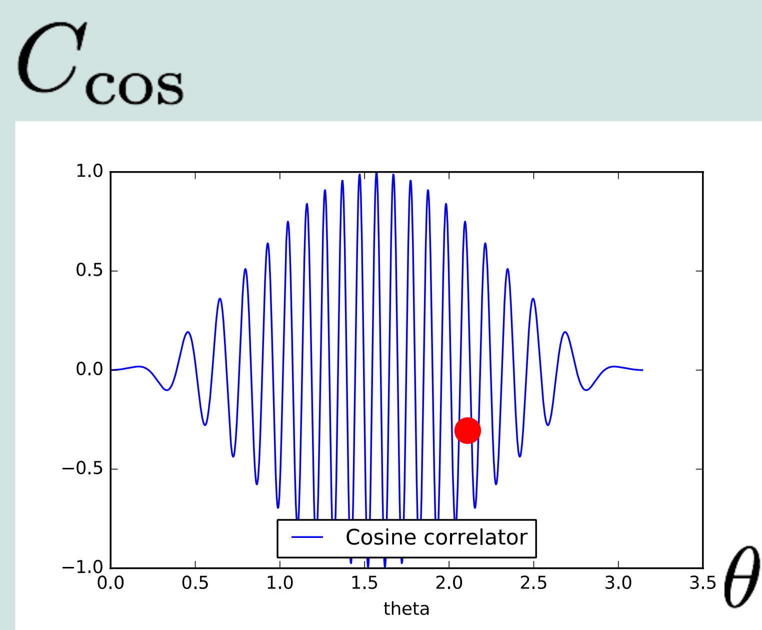
East



West



Assume that the antenna has a non uniform beam pattern



The 2-element interferometer : Untracked source

In optics, the appearance of the interference fringe pattern depends on the location in space.

Here, the strengh of the correlation between the two measured signals will depend on the delay between R_1 and R_2 and therefore, on the direction $\mathbf{s_0}$

The 2-element interferometer : Untracked source

In optics, the appearance of the interference fringe pattern depends on the location in space.

Here, the strengh of the correlation between the two measured signals will depend on the delay between R_1 and R_2 and therefore, on the direction $\mathbf{s_0}$

We can define:

The fringe phase $$\phi = \omega\tau = \frac{\omega}{c}|\mathbf{b}|\cos\theta = \frac{2\pi}{\lambda}|\mathbf{b}|\cos\theta$$

The 2-element interferometer : Untracked source

In optics, the appearance of the interference fringe pattern depends on the location in space.

Here, the strengh of the correlation between the two measured signals will depend on the delay between R_1 and R_2 and therefore, on the direction $\mathbf{s_0}$

We can define:

The fringe phase $$\phi = \omega\tau = \frac{\omega}{c}|\mathbf{b}|\cos\theta = \frac{2\pi}{\lambda}|\mathbf{b}|\cos\theta$$

And its derivative, the fringe rate:

The fringe rate $$|\frac{d\phi}{d\theta}| = \frac{2\pi}{\lambda}|\mathbf{b}\sin\theta| = \frac{2\pi}{T_f}$$

The fringe period T_f

The 2-element interferometer : Untracked source

The correlation of two measured signals can be associated with the angular position of the source with respect to the physical baseline.

If λ is small enough compared to the projected baseline

$$|\mathbf{b}|\sin\theta$$

then the phase of the correlation can precisely track the position of a source.

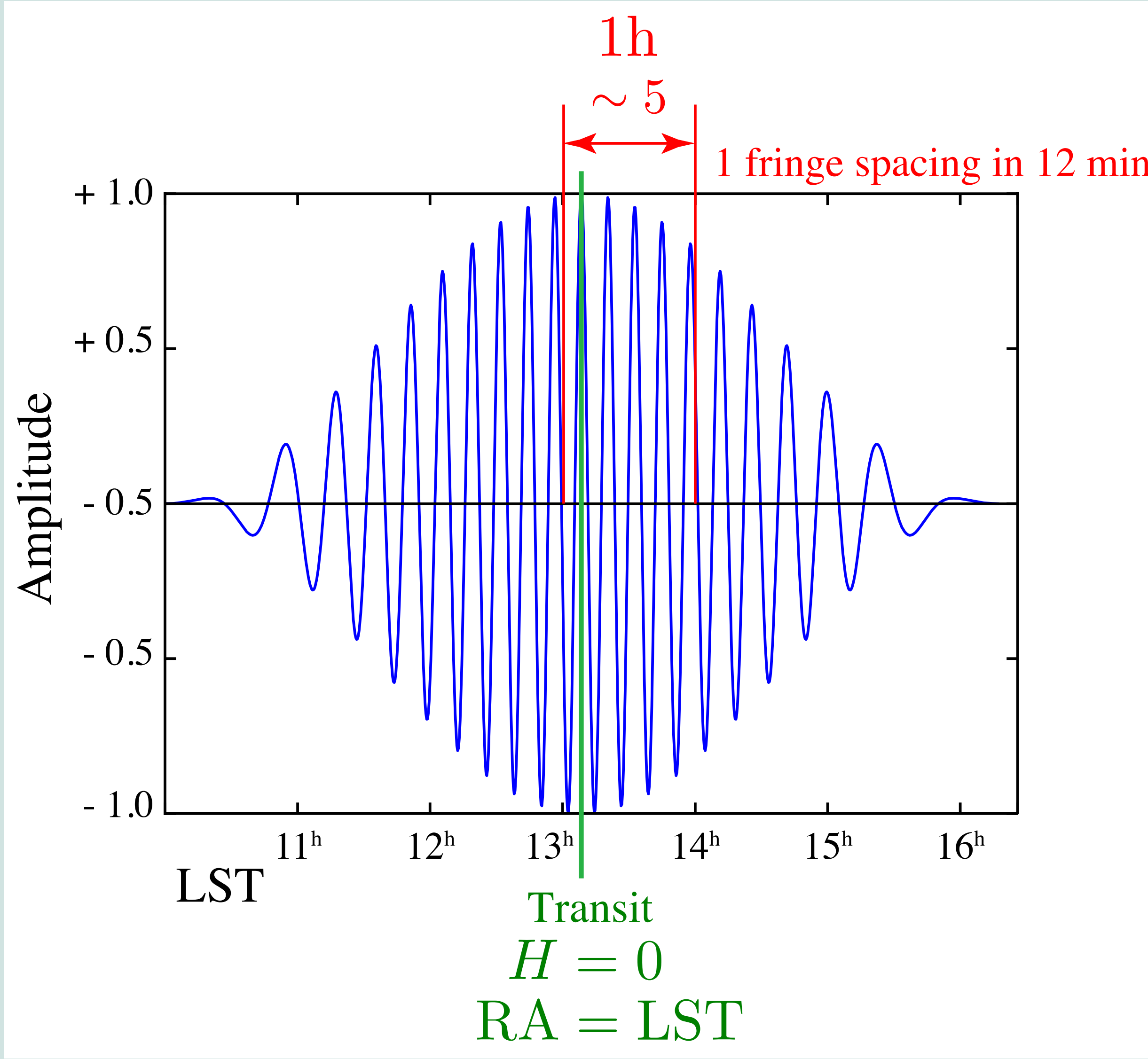
As a consequence, the correlation is sensitive to spatial variations of spatial period T_f.

which means that a 2-element interferometer acts as spatial filter for this spatial frequency.

The 2-element interferometer : Application: position of a source

Estimation of α

High precision measurement of the transit time

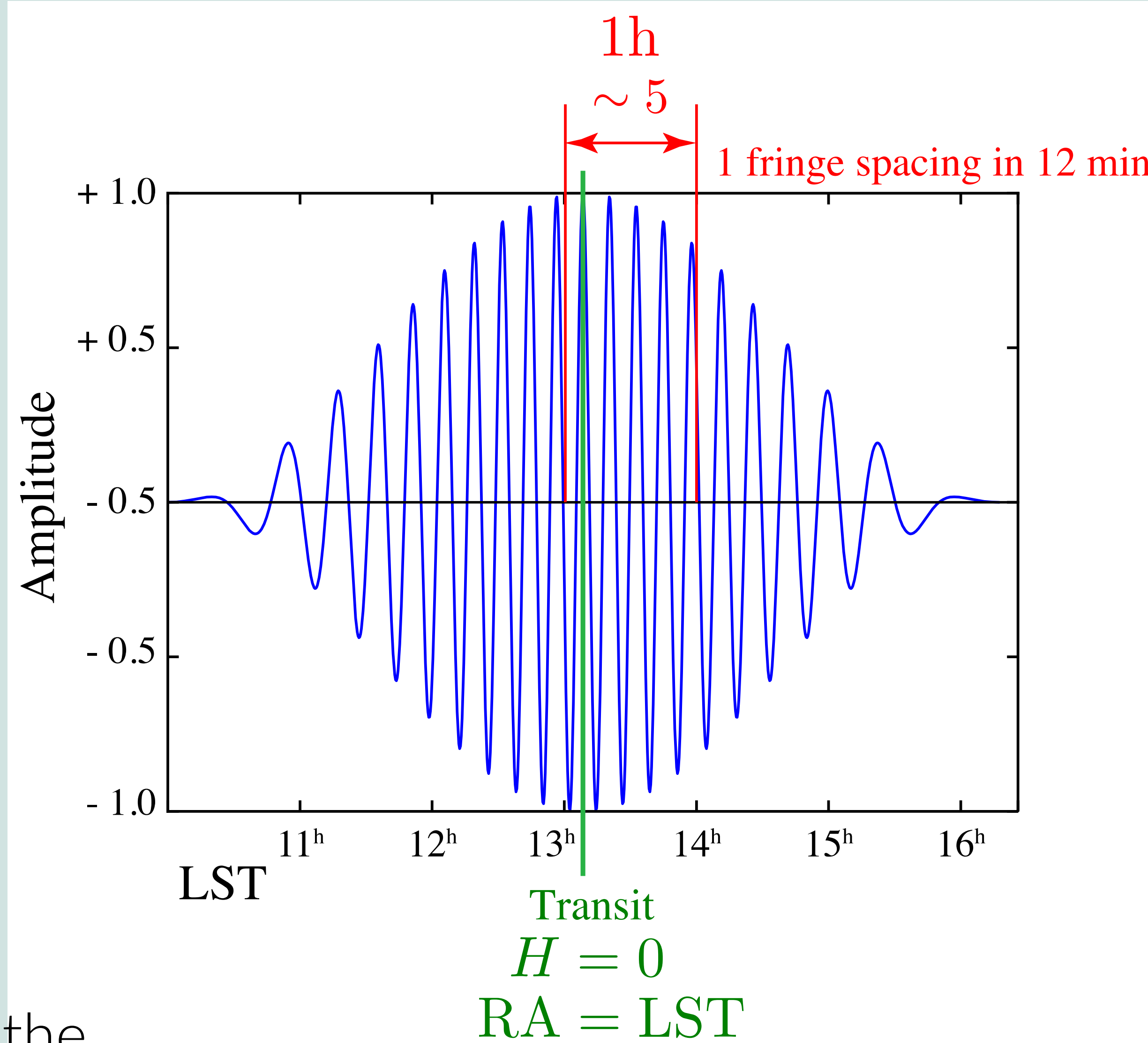


The 2-element interferometer : Application: position of a source

Estimation of α

High precision measurement of the transit time

The transit time can be derived from the maximum of the fringe enveloppe.
It corresponds to the maximum elevation of the source combined to the maximum of the response of the antennas pointing at the local meridian.



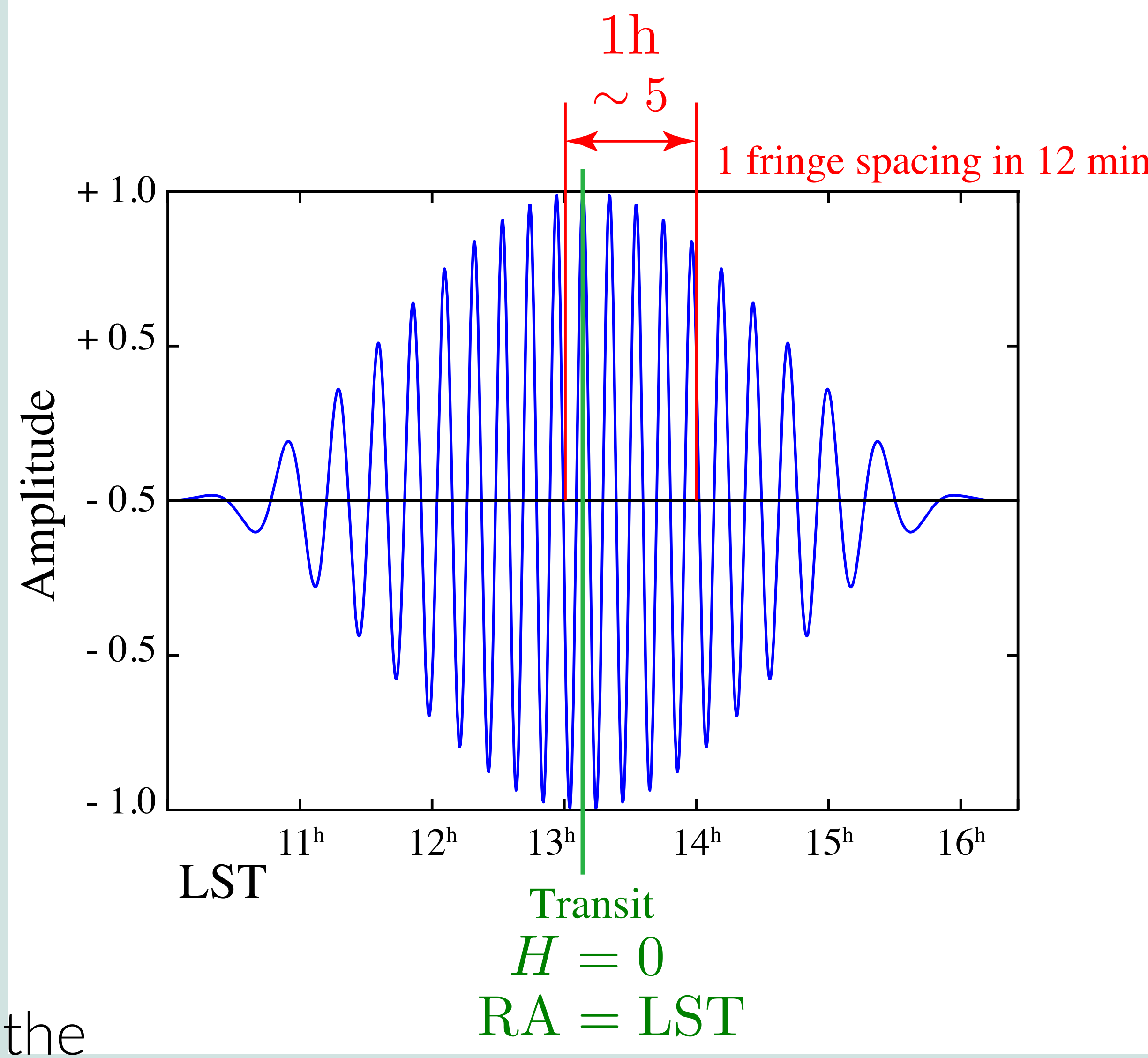
The transit time corresponds to the moment when the LST is equal to the RA

The 2-element interferometer : Application: position of a source

Estimation of α

High precision measurement of the transit time

The transit time can be derived from the maximum of the fringe enveloppe.
It corresponds to the maximum elevation of the source combined to the maximum of the response of the antennas pointing at the local meridian.



The transit time corresponds to the moment when the LST is equal to the RA

Here, we can estimate graphically that $\alpha \sim 13^{\mathrm{h}}07^{\mathrm{m}}$

The 2-element interferometer : Application: position of a source

Estimation of δ

We need to measure the fringe spacing and the fringe rate of the source using the Earth Rotation.

We define the fringe spacing

$$\Delta l_f \sim \frac{\lambda}{|\mathbf{b}|}$$

the measured angular distance on the sky corresponding to one spatial period of the fringe pattern projected on the sky.

We define the fringe rate

$$\frac{d\phi}{d\theta}$$

the time required by the source to cross one spatial period of the fringe pattern.

The 2-element interferometer : Application: position of a source

Estimation of δ

We need to measure the fringe spacing and the fringe rate of the source using the Earth Rotation.

We define the fringe spacing

$$\Delta l_f \sim \frac{\lambda}{|\mathbf{b}|}$$

the measured angular distance on the sky corresponding to one spatial period of the fringe pattern projected on the sky.

We define the fringe rate

$$\frac{d\phi}{d\theta}$$

the time required by the source to cross one spatial period of the fringe pattern.

In our example:

$$\Delta l_f \sim \frac{1}{2864}\ \text{rad} = 0.02\ \text{rad} \approx 1.14^\circ$$ (given)

$\frac{d\phi}{d\theta}$ ~ 12 min (~5 periods are crossed in 1h of observation)